ADAPTIVE CONTROL BY INVERSE MODELING

by B. Widrow,[†] John M. McCool,* and Barry P. Medoff[†]

Stanford University, Stanford, California[†]
Naval Undersea Center, San Diego, California*

Abstract

In a control system, an unknown plant can be made to track an input command signal when this signal is applied to a controller whose transfer function approximates the inverse of the plant's transfer function. The controller output becomes the driving signal for the plant. For real-time applications, the parameters of the controller can be obtained by an adaptive inverse modeling process applied to the plant. If the controller is realized as a transversal filter whose weights are adapted by the LMS algorithm of Widrow and Hoff, stability of the controller can be assured whether modeling the inverse of a minimum phase or a nonminimum phase plant. A random dither signal can be injected to sustain the adaptation process when sufficient ambient signal activity is lacking. Adaptive inverse model control promises simple, economical implementation and robust, predictable behavior. The theory draws from many sources, including the considerable body of knowledge that exists in adaptive signal processing.

I. Introduction

There is a great need for learning-control systems which can adapt to the requirements of plants whose characteristics may be unknown and/or changeable in unknown ways.[1-8] A principal factor that has hampered the development of adaptive controls is the difficulty of dealing with learning processes embedded in feedback loops. Interaction between the feedback of the learning processes and that of the signal flow paths greatly complicates the analysis which is requisite to the design of dependable control systems. In this paper we present a new approach to plant control which circumvents many of the difficulties that have been encountered with previous forms of adaptive control. These techniques have been subjected to preliminary mathematical analysis and they have been successfully tested by means of computer simulated control experiments.

II. Adaptive Filtering

A symbolic representation of an adaptive filter is shown in Fig. 1. The filter has an input, an output, and it requires a special training signal, an input called the "desired response." The "error" is the difference between the desired response and the actual output response. The filter is assumed to be transversal and is adapted by the LMS algorithm of Widrow and Hoff.[9-11]

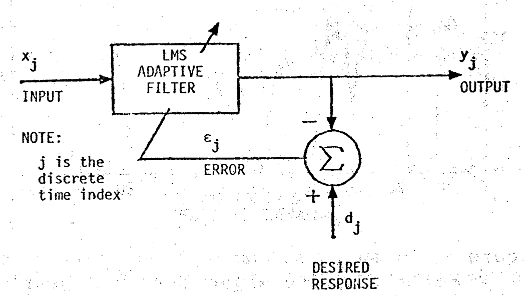

Fig. 1. Symbolic representation of an adaptive transversal filter adapted by the LMS algorithm.

III. Plant Modeling

To illustrate an application of the LMS adaptive filter and to show by example how one obtains an input and a desired response in a control environment, consider the direct modeling of an unknown plant as shown in Fig. 2. When given the same input signal as that of an unknown plant, the adaptive model self-adjusts to cause its

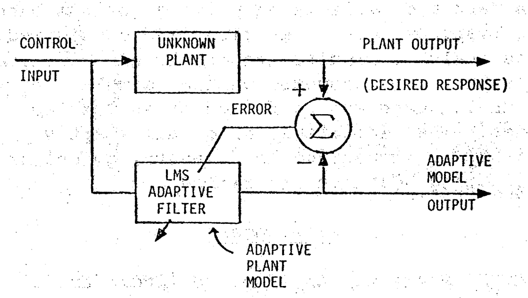

Fig. 2. Modeling an unknown plant by means of an adaptive transversal filter.

This work has been partially supported by NSF Grant No. ENG 78-08526 with Stanford University.

output to be a best least squares fit to the actual plant output. The unknown plant may have both poles and zeros, but the adaptive transversal filter can only realize "zeros." (The word zeros is in quotes because the adaptive filter is time variable and does not strictly have a transfer function. In a quasi-static sense, the adaptive filter can be thought to have "instantaneous zeros" corresponding to the zeros that would exist if the weights were frozen at their instantaneous values.) With a sufficient number of weights, an adaptive transversal filter can achieve a close fit to an unknown plant having many poles and zeros.

IV. Plant Inverse Modeling

The inverse model of the unknown plant could be formed as shown in Fig. 3. The adaptive filter input is the plant output. The filter is adapted to cause its output to be a best least squares fit to the plant input. A close fit implies that the cascade of the unknown plant and the LMS filter have a "transfer function" of essentially unit value. Close fits have been achieved by adaptive transversal inverse filters even when the unknown plant had many poles and zeros.

V. Inverse Modeling Nonminimum Phase Plants

If the plant itself is stable, all of its poles lie in the left half of the s-plane. But some of its zeros could lie in the right half plane, and then the plant would be nonminimum phase. The inverse of the minimum phase plant would have all of its poles in the left half plane, and there would be no problem with stability of the inverse. The nonminimum phase plant would have poles in the right half plane and stability of the inverse would be an important issue. However, it can be shown that stable inverses for nonminimum phase plants could always be constructed if one were permitted noncausal two-sided impulse responses. Furthermore, with suitable time delays, causal approximations to delayed versions of noncausal impulse responses are realizable. Thus, by allowing a delay in the modeling process (as illustrated in Fig. 3), one can obtain approximate delayed inverse models to minimum phase and nonminimum phase plants. It is not necessary to know *a priori* whether the plant is or is not minimum phase. However, some knowledge of plant characteristics would be helpful when choosing the delay Δ and the length of the transversal filter used for inverse modeling.

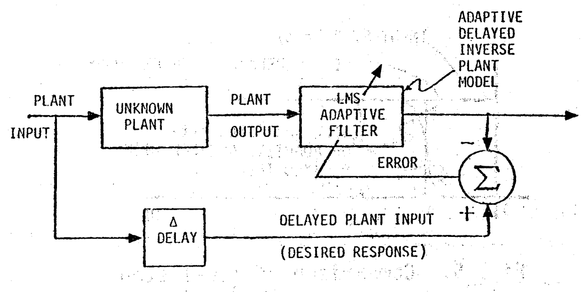

Fig. 3. Delayed inverse modeling of an unknown plant.

VI. The Adaptive Inverse Model Control Scheme

Using a stable delayed inverse, control is accomplished as illustrated in Fig. 4. The controller is a copy of the inverse model. The reference command, the desired output for the plant, is applied as an input to the controller. The controller output is the driving function for the plant. If the controller were an exact delayed plant inverse, the plant output, assuming no noise, would be an exact copy of the input reference command, but delayed, i.e.,

$$c(t) = r(t-\Delta) \quad .$$

A step change in the reference command would cause a step change in the plant output after a delay of Δ seconds. The idea is illustrated by Fig. 5. If the inverse model is imperfect but a good

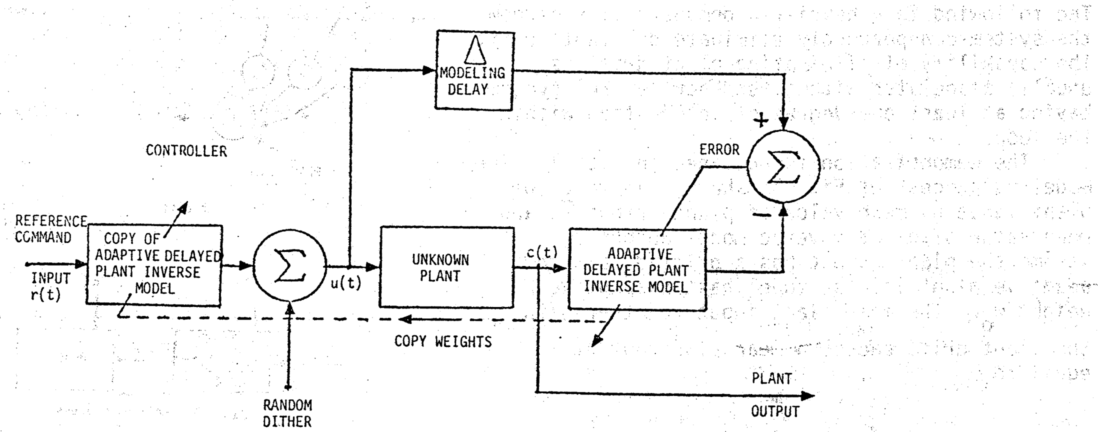

Fig. 4. Adaptive inverse model control system.

approximation to the plant inverse, then the step response c(t) might appear as illustrated. The ideal delayed step response is superimposed. A typical step response of a simple feedback control system is sketched in Fig. 6. It is interesting to compare this response with that of the inverse model control scheme.

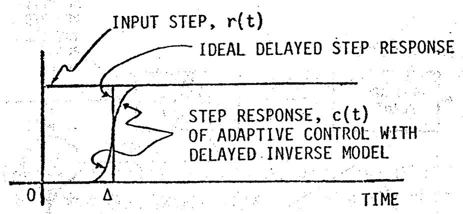

Fig. 5. Comparison of ideal step response vs. adaptive delayed inverse model control system step response.

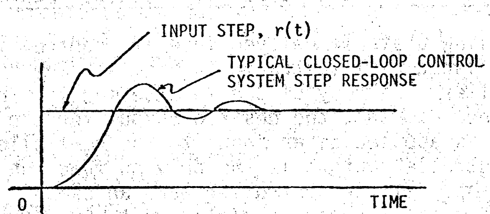

Fig. 6. Step response of a conventional closed-loop control system.

VII. Effects of Plant Drift

The control system of Fig. 4 appears to be open loop. Actually it is a closed-loop system, with the loop closed through the adaptive process. The following is a heuristic demonstration of how the system can perfectly eliminate dc plant drift. The capability of eliminating plant drift is usually associated with a feedback control system having at least one degree of integration within the loop.

The demonstration is outlined in Fig. 7. The modeling process of Fig. 7a shows mean value of plant input $\bar{u}$, mean value of plant output $\bar{r}$, and mean value of plant inverse model output $\bar{s}$. Assume the plant output has a drift d, and the adaptive plant inverse model has an adaptive bias weight w_0. The mean plant input together with the plant drift causes a mean plant output $\bar{r}$ equal to

$$\bar{r} = d + \bar{u} \sum_{i=1}^{m} a_i \quad .$$

The plant is being described here in terms of a discrete impulse response comprising the set of coefficients a_i. The structure of the plant (with drift d) is shown in Fig. 7b. The structure of the adaptive inverse model (with its bias weight w_0) is shown in Fig. 7c. Referring again to Fig. 7a, the mean inverse plant model output s is seen to be

$$\bar{s} = w_0 + \bar{r} \sum_{k=1}^{n} w_k \quad .$$

Since the adaptive process minimizes MSE (mean square error), the bias weight w_0 will be automatically adjusted so that the error of inverse modeling will unbiased (any bias will needlessly increase MSE). Therefore

$$\bar{s} = \bar{u} \quad .$$

Next refer to Fig. 7d, which shows the control process. Let the mean input to the inverse plant model be $\bar{r}$. The inverse model output will be $\bar{s} = \bar{u}$ which is applied to the plant for control. But we have already noted that $\bar{u}$ into the plant (experiencing drift d) causes a mean plant output of $\bar{r}$. We therefore conclude that an input reference command mean value of $\bar{r}$ causes a mean plant output response of $\bar{r}$, regardless of the amount of plant drift d.

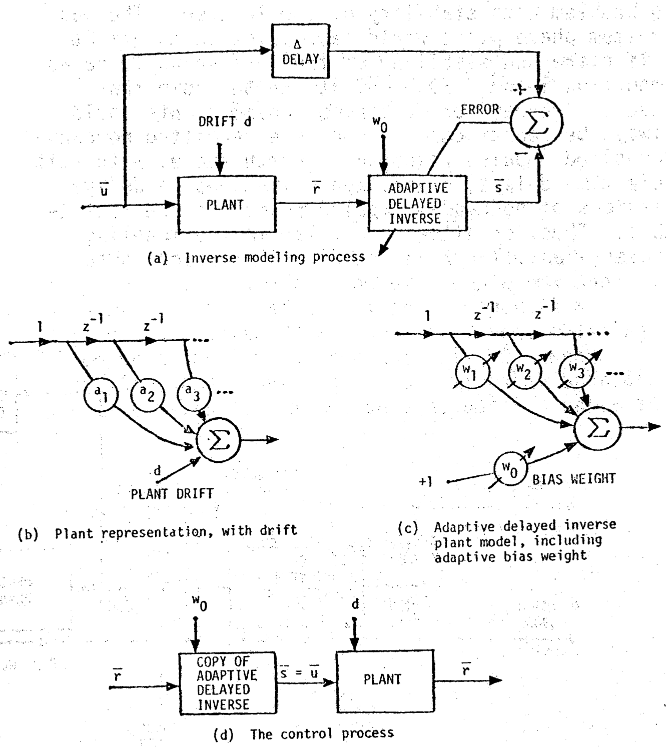

Fig. 7. Demonstration of control of plant output mean (dc level) in the presence of unknown plant drift.

A number of questions remain to be answered. For example, how fast can the drift d change without causing plant output bias error? How fast can the mean of the input reference command change yet have the mean plant output follow without significant error? What is the effect of dynamic inverse modeling errors upon system performance?

In a control application, the price to be paid for a noncausal inverse is a delayed control response. In many cases however, this delay would be inevitable because of dynamic delays in the plant. A controller using the delayed inverse model would have a precisely predictable delay in the control response. Furthermore, the system step response would have very little ringing or overshoot. This is confirmed in a preliminary way by the computer simulation results of the next section.

VIII. Simulation of an Adaptive Inverse Model Control System

The system that was simulated is shown in Fig. 4. A small amplitude unbiased random dither signal is introduced into u(t), the plant input, in order to insure that the adaptive modeling process will continue to keep the inverse model current, regardless of the ambient control signals present. The dither is necessary in many cases to keep the adaptive process "alive," but it does disturb the plant and introduce noise into the output.

The simulations used both a discrete-time plant and a discrete-time controller. Many combinations of plant pole and zero locations were tried. Typical results are shown here. A plant having two poles and no zeros was simulated. Pole locations in the z-plane of this plant (#1) are shown in Fig. 8. Its step response is shown in Fig. 9.

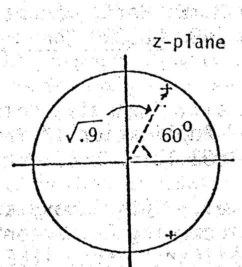

Fig. 8. Location of poles of Plant #1.

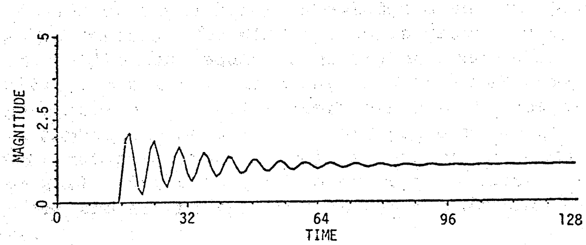

Fig. 9. Step response of Plant #1.

The inverse of this plant had two zeros and was easily realized by the adaptive transversal filter which had available thirty-two weights. The inverse impulse response is plotted in Fig. 10. Using this inverse as a controller, the step response of the entire system is precise, as shown in Fig. 11.

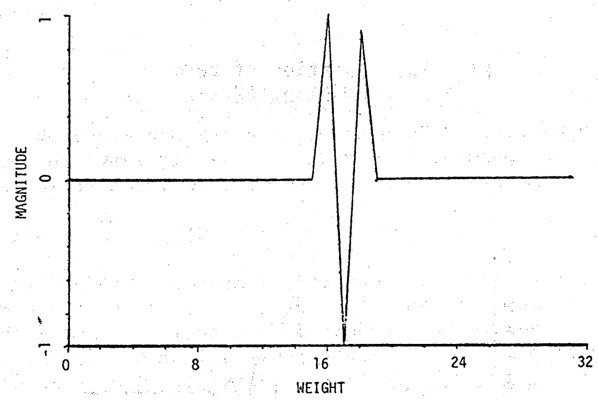

Fig. 10. Delayed impulse response of inverse of Plant #1.

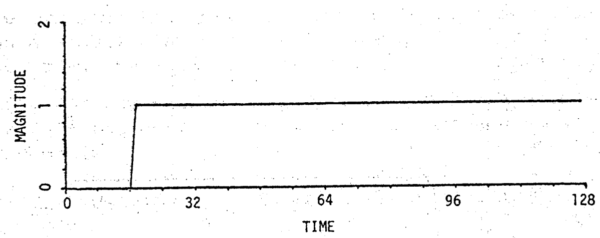

Fig. 11. For Plant #1, control system step response.

A potentially more difficult case is that of controlling plant #2 which has no poles and only one zero located as shown in Fig. 12. This is a nonminimum phase plant. The delayed inverse was formed and used as a controller. The inverse's impulse response is shown in Fig. 13; the step response of the resulting control system is shown in Fig. 14. In this case there was no real difficulty in controlling a nonminimum phase plant.

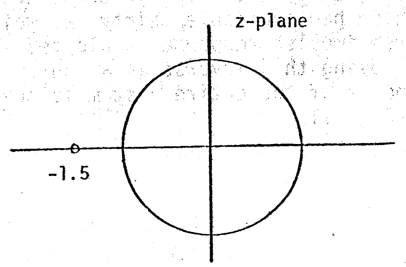

Fig. 12. Location of zero of Plant #2.

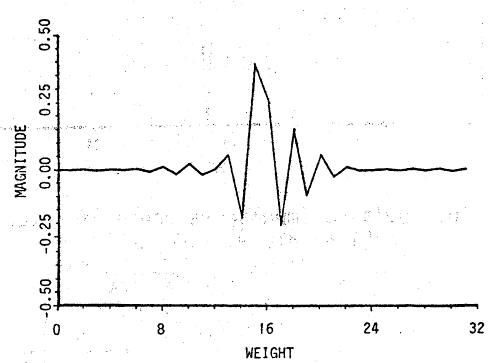

Fig. 13. Delayed inverse impulse response for Plant #2.

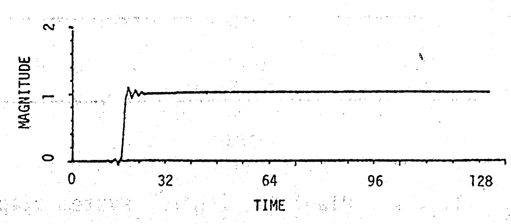

Fig. 14. For Plant #2, control system response.

Many other cases have been successfully tried. Tightness of control depends on closeness of fit between the true inverse and the inverse realized by the adaptive transversal filter. Dither signal spectral characteristics and number of weights used in the inverse have much to do with the closeness of fit. Work on adaptive inverse model control is continuing and will be reported in the future.

References

[1] K. S. Fu, "Learning Control Systems and Intelligent Control System: an Intersection of Artificial Intelligence and Automatic Control," IEEE Transactions on Automatic Control, February 1971.

[2] C. Leondes, Modern Control Systems Theory, McGraw-Hill, New York, 1965.

[3] F. D. Powell, "Predictive Adaptive Control," IEEE Transactions on Automatic Control, October 1969.

[4] E. Tse and M. Athans, "Adaptive Stochastic Control for a Class of Linear Systems," IEEE Transactions on Automatic Control, February 1972, pp. 38-51.

[5] K. Nakamura and Y. Yoshida, "Learning Dual Control Under Complete State Information," a paper presented at NSF Workshop on Learning System Theory and its Applications, October 18-20, 1973, in Gainesville, Florida.

[6] K. J. Astrom and B. Wittenmark, "On Self-tuning Regulators," Automatica, Vol. 9, No. 2, March 1973.

[7] I. D. Landau, "Unbiased Recursive Identification Using Model Reference Adaptive Techniques," IEEE Transactions on Automatic Control, Vol. 21, April 1976.

[8] J. M. Martin-Sanchez, "A New Solution to Adaptive Control," Proc. IEEE, Vol. 64, No. 8, August 1976.

[9] B. Widrow and M. E. Hoff, "Adaptive Switching Circuits," in 1960 WESCON Conv. Rec., pt. 4, pp. 96-140.

[10] B. Widrow, et al., "Adaptive Noise Cancelling: Principles and Applications," Proc. IEEE, Vol. 63, pp. 1692-1716, December 1975.

[11] B. Widrow, et al., "Stationary and Nonstationary Learning Characteristics of the LMS Adaptive Filter," Proc. IEEE, Vol. 64 pp. 1151-1162, August 1976.