A LINEAR PHASE ADAPTIVE FILTER

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Abstract - A general purpose linear phase adaptive filter is presented. The filter allows the use of either positive or negative impulse response symmetry to invoke a linear phase response. It is shown that for a particular choice of inputs, a linear phase adaptive line enhancer can be realized.

I. Introduction

Adaptive methods have been used to construct a number of diverse signal processing structures. A majority of these structures are based on the adaptive linear combiner shown in Fig. 1. When the X-vector input for the linear combiner is generated as shown in Fig. 2 , a general purpose FIR adaptive filter results. This adaptive filter is the basic building block for signal processing techniques such as noise canceling, array processing, line enhancement, channel equalization, spectral estimation, adaptive control, and system identification [1-5].

Often in the above schemes it is important that the output of the adaptive filter maintain the relative phase relationships of its spectral components, thus requiring that the filter be linear phase. In [6], Friedlander and Morf describe an adaptive linear phase smoothing filter which estimates a signal based on advanced and delayed samples of that signal. A common use of such a structure is the adaptive line enhancer [1] which we will show to be a special case of the general linear phase adaptive filter. This correspondence details a general purpose linear phase adaptive filter originally presented by Widrow et al in [7] and discusses its specialization to a linear phase predictor, hereafter referred to as a linear phase adaptive line enhancer (ALE). A simulation of the linear phase adaptive filter in a channel equalization application is also presented.

II. An Adaptive Linear Combiner

The adaptive linear combiner shown in Fig. 1 uses a linear combination of n inputs to form a "best" estimate of some desired response input.

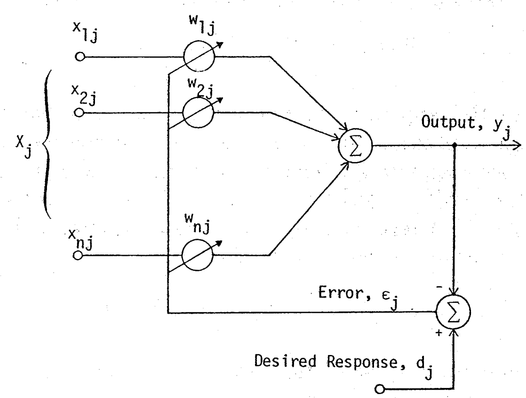

Fig. 1 - An adaptive linear combiner.

Let the n input signals define an X-vector given by

$$X_j^T = [\, x_{1j}\ x_{2j}\ \cdots\ x_{nj}\,] \quad . \tag{1}$$

Also let the weighting vector be defined as

$$W_j^T = [\, w_{1j}\ w_{2j}\ \cdots\ w_{nj}\,] \quad . \tag{2}$$

Then the linear combiner output is

$$y_j = \sum_{k=1}^{n} w_{kj} \cdot x_{kj} = W_j^T \cdot X_j \quad . \tag{3}$$

The error between this output and the desired response input is

$$\epsilon_j = d_j - y_j = d_j - W_j^T \cdot X_j \quad . \tag{4}$$

The best estimate of d_j is found by minimizing the mean square of the error ϵ_j.

One of the most popular methods for iteratively minimizing the mean square error is the LMS algorithm [8]. This gradient-descent type algorithm updates the weight vector using an approximate gradient according to,

$$W_{j+1} = W_j - \mu \cdot \hat{\nabla}_j \quad . \tag{5}$$

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The estimated gradient is the instantaneous gradient of the squared error and is given by

$$\hat{\nabla}_j = -2 \cdot \epsilon_j \cdot X_j \quad . \tag{6}$$

Eqs. (4-6) form the LMS algorithm. The stability and convergence of this algorithm has been extensively studied by Widrow in [8] and more recently by Horowitz and Senne in [9].

Another method for updating the weight vector is the well-known recursive least squares (RLS) algorithm [10,11]. It can be shown that this algorithm is equivalent to a Newton-type LMS algorithm [12] of the form,

$$W_{j+1} = W_j - \alpha \cdot \hat{R}_{j+1}^{-1} \cdot \hat{\nabla}_j \quad , \tag{7}$$

where the approximate gradient vector is premultiplied by an approximation of the inverse correlation matrix. The approximate gradient vector is the same as that used in the LMS algorithm. The approximate correlation matrix is determined by taking an exponentially windowed sample average of the input data,

$$\hat{R}_{j+1} = (1-\alpha) \cdot \hat{R}_j + \alpha \cdot X_j \cdot X_j^T \quad . \tag{8}$$

To find the inverse of this estimate, the matrix inversion lemma [11] is applied to eq. (8) giving,

$$\hat{R}_{j+1}^{-1} = \frac{1}{1-\alpha} \cdot \left(\hat{R}_j^{-1} - \frac{\alpha \cdot \hat{R}_j^{-1} \cdot X_j \cdot X_j^T \cdot \hat{R}_j^{-1}}{(1-\alpha) + \alpha \cdot X_j^T \cdot \hat{R}_j^{-1} \cdot X_j} \right) . \tag{9}$$

Eqs. (6,7,9) determine the RLS algorithm.

The RLS algorithm is computationally more complex than the LMS algorithm but offers an improvement in convergence rate when the eigenvalue disparity of the input correlation matrix is large [8]. For many applications the simple LMS algorithm is the preferred method.

III. A Linear Phase Adaptive Filter

Fig. 2 shows the linear combiner X-vector input required to realize an n-tap FIR adaptive filter.

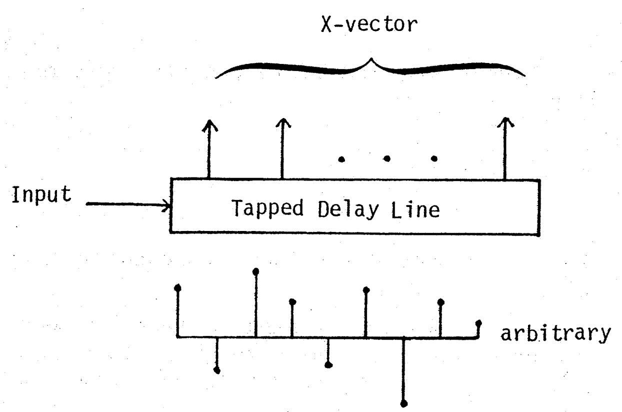

Fig. 2 - Unconstrained linear combiner input with sample impulse response.

When this X-vector is used with the adaptive linear combiner of Fig. 1, a general purpose adaptive filter results. Since the filter impulse response is not symmetrically constrained, its phase response will not necessarily be linear.

Fig. 3 shows the n-element X-vector input required to realize a positive symmetry linear phase adaptive filter.

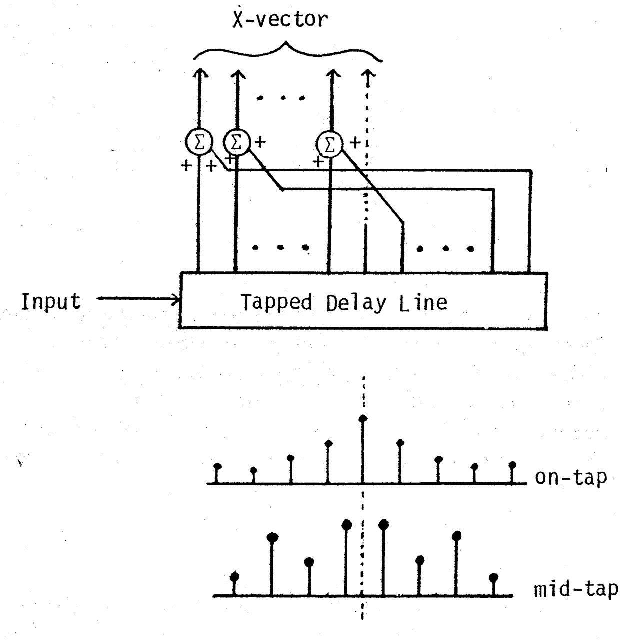

Fig. 3 - Constrained linear phase positive symmetry linear combiner input with sample impulse response.

Two subcases of this positive symmetry exist [13]. The first is called the on-tap symmetry case since the filter coefficients are symmetric about the center tap of the tapped delay line (TDL). In this case, the TDL has an odd number of taps. Each of the first $n-1$ X-vector elements is the *sum* of two TDL outputs chosen symmetrically about the center tap. The center tap output is used as the last element of the X-vector. The other case is called mid-tap symmetric since the filter coefficients are symmetric about a point midway between the two center taps of the TDL. Here, the TDL has an even number of taps. Again, each X-vector element is the sum of two TDL outputs, with the sums chosen symmetrically about the mid-tap center of the TDL.

Fig. 4 shows the n-element X-vector required to realize a negative symmetry linear phase adaptive filter.

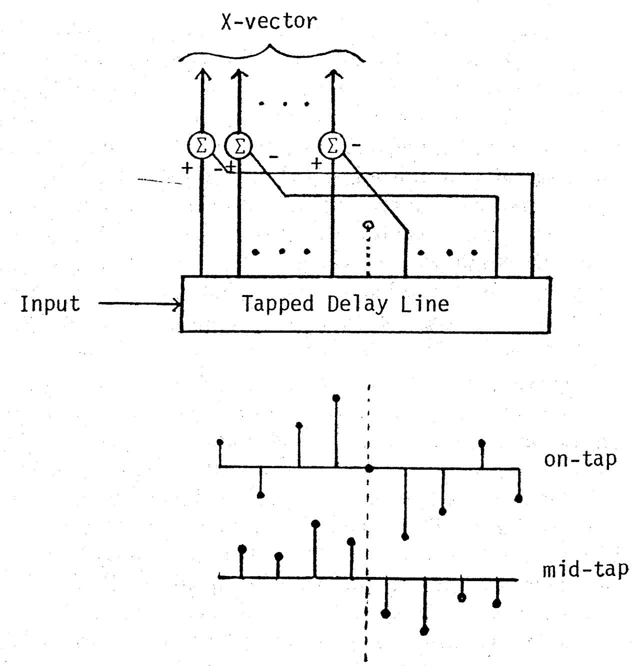

Fig. 4 - Constrained linear phase negative symmetry linear combiner input with sample impulse response.

In the on-tap negative symmetry case, each X-vector element is the ***difference*** between two TDL outputs chosen symmetrically about the center of the TDL. Note that the center tap output of the TDL is not used. The resulting linear phase filter will have an impulse response of length $2n+1$, with the center coefficient set to zero. In the mid-tap negative symmetry case, each X-vector element is the difference between the two TDL outputs chosen symmetrically about the mid-tap center of the TDL. The filter impulse response will be of length $2n$.

It is easily seen that the transfer function of a linear phase FIR [13] filter with positive symmetry can be written as,

$$H_e(j\omega) = R_e(\omega)\cdot e^{-j\frac{m-1}{2}\omega}, \tag{10}$$

where R_e is real and m is length of the filter impulse response. Similarly, the transfer function of a linear phase FIR filter with negative symmetry can be written as,

$$H_o(j\omega) = j\cdot R_o(\omega)\cdot e^{-j\frac{m-1}{2}\omega} \tag{11}$$

Note that the phase response of the negative symmetry filter is the sum of an exact 90^o phase shift plus a linear phase term.

Fig. 5a shows both the X-vector input and the desired response input required to realize an adaptive line enhancer with a Δ-step decorrelation delay [1].

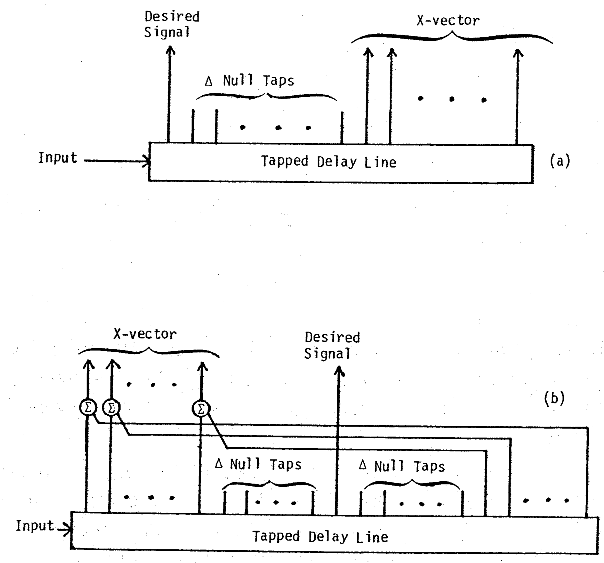

Fig. 5 - Linear combiner inputs required for Adaptive Line Enhancer (ALE).
(a) unconstrained ALE (b) constrained linear phase ALE.

Forming the X-vector as shown in Fig. 5b results in a linear phase adaptive line enhancer. Each X-vector element is the sum of two TDL outputs chosen symmetrically about the center tap of the TDL. The center output of the TDL is used as the linear combiner's desired response. The resulting adaptive structure will estimate the signal at the center-tap output of the TDL, based on delayed and advanced samples of that signal.

The linear phase adaptive smoother discussed by Friedlander and Morf [6] is a special case of the linear phase ALE with a decorrelation delay of one. In their paper they show that for a stationary stochastic input, the Wiener solution of the smoothing filter will be linear phase. Thus, using constrained linear phase to reduce the number of computations will not sacrifice the quality of the solution. It has also been shown that using a smoothing filter rather than a predicting filter can improve the convergence rate of the LMS adaptive algorithm [14], further motivating the use of the linear phase ALE.

IV. Simulation

In this section we present a simulation of the linear phase adaptive filter applied to baseband channel equalization. A block diagram of the simulation is shown in Fig. 6.

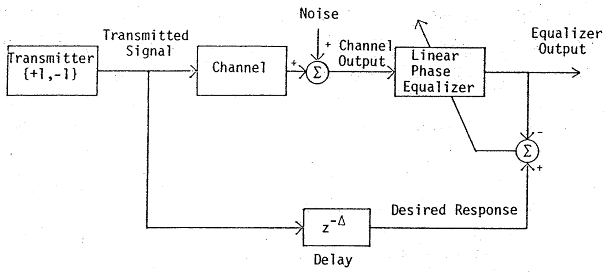

Fig. 6 - Block diagram of a linear phase baseband channel equalization simulation.

The system consists of a transmitter, an imperfect but linear phase transmission path (the channel), and a linear phase adaptive equalizer. The goal of the equalizer is to "undo" the signal distortion caused by the imperfect channel.

It should be noted that most channels encountered in practice will not be linear phase. The purpose of this simulation is not necessarily to show a practical application of the linear phase adaptive filter, but to demonstrate its feasibility.

For this simulation, the channel is a linear phase filter with a 30 dB rolloff at both high and low frequencies, modeling the limited frequency response of a typical transmission path. The impulse response of the simulated channel is
{.014, -.002, -.073, .010, .233, -.021, -.463, .024, .667, .024, -.463, -.021, .233, .010, -.073, -.002, .014}.
Uniformly distributed white noise with variance 30 dB below the signal is added to the channel output, simulating the effects of a noise corrupted channel.

The transmitter signal consists of the random {+ 1,-1} sequence shown in Fig. 7a.

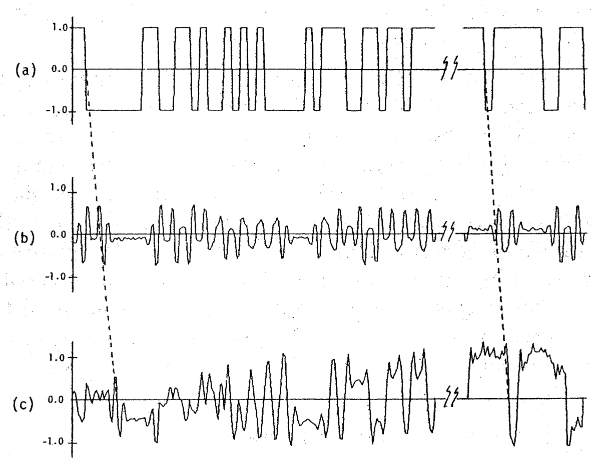

Fig. 7 - Time waveforms of linear phase adaptive channel equalizer simulation using LMS algorithm.
(a) channel input (b) channel output (c) equalizer output

From Fig. 7b, it is evident that the limited frequency response of the channel has caused the channel output to be quite distorted. When the channel is close to linear phase, as is the case in this simulation, it can be shown that a linear phase equalizer will perform well in restoring the original transmitted signal shape. For this experiment, a positive on-tap symmetric linear phase adaptive filter was used as the equalizer. The total impulse response length was 9, which translates into a 5-weight adaptive linear combiner. Note that the linear phase constraint reduces the number of adaptive weights by a factor of 2. For this experiment, μ was set to .002.

In a typical adaptive equalizer application, the equalizer is adjusted by generating a synchronized training signal at both the transmitter and receiver locations. This signal is used directly as the transmitter signal and in a delayed form as the desired response signal for the adaptive equalizer. The delay, shown in the bottom half of Fig. 6, is chosen to be equal to the total delay through the cascade of the channel and the equalizer. In this case the delay was chosen to be 13 sample instants. Fig. 7c shows 240 iterations of the start-up transient of the adaptive equalizer output, along with 60 iterations of the equalizer output after convergence. The sloped dotted lines intersect equivalent points of the signals, illustrating the delays due to the channel and equalizer. After about 120 samples, the equalizer has improved the signal to the point where a simple zero-crossing algorithm could restore the transmitted signal exactly. After the equalizer has converged, its weights are fixed and the equalized channel can be used for normal data transmission.

V. Conclusion

In this paper we have shown that a linear phase adaptive filter can be realized by supplying the well-known adaptive linear combiner with the proper choice of X-vector input. A new adaptive algorithm is not required. Although simple in derivation, the linear phase adaptive filter presented in this paper is expected to be a popular addition to the existing set of tools now used by the signal processing community.

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